

Exercise 33

Find equations of the tangent line and normal line to the given curve at the specified point.

$$y = 2xe^x, \quad (0, 0)$$

Solution

Start by finding the slope of y at $x = 0$. Evaluate the derivative using the product rule.

$$\begin{aligned} y' &= \frac{d}{dx}(2xe^x) \\ &= \left[\frac{d}{dx}(2x) \right] (e^x) + (2x) \left[\frac{d}{dx}(e^x) \right] \\ &= (2)(e^x) + (2x)(e^x) \\ &= 2(1+x)e^x \end{aligned}$$

Evaluate it at $x = 0$.

$$y'(0) = 2$$

Therefore, the equation of the tangent line with slope 2 and the equation of the normal line with slope $-1/2$ that go through $(0, 0)$ are respectively

$$y - 0 = 2(x - 0) \quad \text{and} \quad y - 0 = -\frac{1}{2}(x - 0).$$

